ELIZADE UNIVERSITY, ILARA-MOKIN, ONDO STATE

FACULTY OF ENGINEERING

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

SEMESTER II EXAMINATION, 2015/2016 ACADEMIC SESSION

COURSE TITLE: NUMERICAL METHODS

COURSE CODE: EEE 312

EXAMINATION DATE: 21^{ST} JULY, 2016

COURSE LECTURER: DR R. O. Alli-Oke

HOD's SIGNATURE

TIME ALLOWED: 3 HRS

INSTRUCTIONS:

1. ANSWER ALL QUESTIONS **(TOTAL OF 3 QUESTIONS)**
2. SEVERE PENALTIES APPLY FOR MISCONDUCT, CHEATING, POSSESSION OF UNAUTHORIZED MATERIALS DURING EXAM.
3. YOU ARE **NOT** ALLOWED TO BORROW CALCULATORS AND ANY OTHER WRITING MATERIALS DURING THE EXAMINATION.

1)

a) Compute a real root (correct to 4 decimal places) of $f(x) = x^2 - 3$. Let $\varepsilon_{width} = \varepsilon_{abs} = 0.01$ and start with interval [1.65, 1.75]. Clearly show your workings for only the first iteration. The tabled results should display the following - n, a, b, $f(a)$, $f(b)$, c, $f(c)$, $f(a).f(c)$, updated $|b - a|$ - where a, b and c are the end points and mid-points respectively. The algorithm should be stopped when the interval width less than ε_{width} and $f(c)$ is less than ε_{abs}. **(8 marks)**

b) The "complex factorization theorem" states that every polynomial that is not identically zero has exactly n roots (counting multiplicity). These roots may be real or complex. Prove that every polynomial having real coefficients can be factored into a product of linear and quadratic factors with real coefficients. *Hint: You may use the complex conjugate-root theorem.* **(5 marks**

c) Given a polynomial $P(x)$ of degree n as follows

$$P(x) = a_{n+1}x^n + a_n x^{n-1} + \cdots + a_{k+1}x^k + \cdots a_3 x^2 + a_2 x^1 + a_1$$

Then $P(x)$ can be expressed as $P(x) = ((x^2 - rx - s)Q(x)) + R(x)$,

where $R(x)$ is the remainder, a binomial of degree 1, $b_2(x - r) + b_1$;

$Q(x)$ is the quotient, a polynomial of degree $n - 2$, $b_{n+1}x^{n-2} + b_n x^{n-3} \ldots + b_{k+3}x^k + \cdots b_5 x^2 + b_4 x^1 + b_3$;

and the coefficients are given by

$b_{n+1} = a_{n+1}$,

$b_n = a_n + rb_{n+1}$,

$b_{k+1} = a_{k+1} + rb_{k+2} + sb_{k+3}$ for $k = n-2, n-3, \ldots 2, 1, 0$

Use Bairstow's method to determine the approximate complex roots (correct to 4 decimal places) of the polynomial $P(x)$ where $P(x) = x^3 - 3x^2 + 4x - 2$. Let the initial values of r and s be $r_1 = 1.9$ and $s_1 = -2.2$ respectively. Clearly show your workings for only the first iteration. The tabled results should display the following - r, s, Δr, Δs, ε_r, ε_s - where $\varepsilon_r, \varepsilon_s$ are the *relative approximate error* in the estimates of (r, s). The algorithm should be stopped when both *relative approximate error* in the estimates of (r, s) are less than the precision value of 2×10^{-2}. *Hint: obtain the roots from* $Q(x)$. **(14 marks)**

2)

a) Define the interpolation problem. **(5 marks)**. State 3 bases for constructing interpolating polynomials. **(3 marks)**.

b) Using Lagrange Basis, determine an approximate function,

$$p(x) = \sum_{i=1}^{n} a_i L_i(x) = \sum_{i=1}^{n} a_i \prod_{\substack{j=1 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j},$$

that interpolates the following data points,

x_i	0	1	−1	2	−2
f_i	−5	−3	−15	39	−9

(8 marks)

c) State, in no more than 2 sentences, the difference between Newton-Cotes quadrature rules and Gaussian quadrature rules. Show that the Simpson's $1/3$ quadrature rule over an interval $[a, b]$ is given by,

$$\int_a^b f(x) \approx \frac{b-a}{6}\left(f(a) + 4f\left(\frac{b+a}{2}\right) + f(b)\right)$$

Hint: Use a second-order Lagrange interpolating polynomial to approximate $f(x)$. **(9 marks)**

3)

a) Consider the following system of linear equations, $Ax = b$ where x is the vector of unknown variables and A, b are of appropriate dimensions. State clearly, [illegible] the difference between an under-determined and an over-determined system of linear equations? ([illegible] marks)

b) When is a system of linear equations inconsistent? ([illegible] marks)

c) Let P, L, U be a permutation matrix, lower triangular matrix and an upper triangular matrix respectively. Let $A =$ [illegible]

Use LU factorization to show that $PA = LU$ for the given A matrix. Perform partial (row) pivoting only when the pivot element is zero. ([illegible] marks)